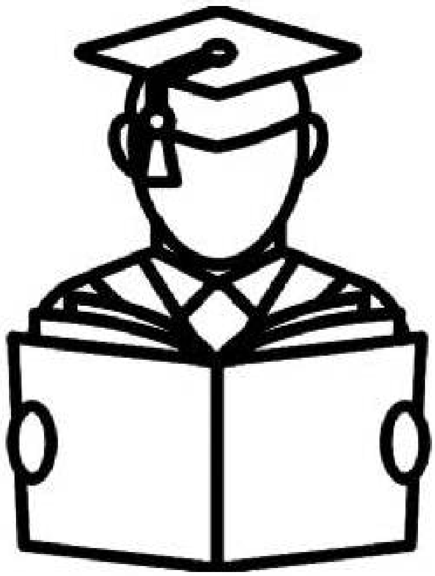
चौधरी PHOTOSTAT

"I don't love studying. I hate studying. I like learning. Learning is beautiful."



"An investment in knowledge pays the best interest."

Hi, My Name is

Chemical Engineering for GATE/IES

(*MADE EASY*)

Matrices

Properties of determinants :

(i) If two rows or columns of matrix are identical, then the determinant is zero

$$\Delta = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & 1 & 2 \end{vmatrix} = 0$$

(ii) If two rows or columns of matrix are interchanged then the sign of determinont changes.

$$\Delta = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} \qquad -\Delta = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 1 & 3 & 1 \end{vmatrix}$$

(iii) If three rows or columns of matrix are interchanged then the sign of determinant is unaltered.

$$\Delta = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

(iv) In the determinont of matrix if any column containing sum or difference of two elements then it can be split into sum or difference of two determinonts.

$$\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Delta = ad - bc$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\Delta = a_{11}\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

operations for solving matrix are always less than one the order of matrix.

Upper and lower triangular matrix :

If all the elements above principal diagonal are zero then its said to be lower triangular matrix and if all elements below the principal diagonal are zero it is upper triangular matrix.

$$\text{Lower triangular matrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix} = 18$$

$$\text{Upper triangular matrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} = 24$$

Note:

If is matrix is either lower triangular or upper triangular determinant is product of principal diagonal elements.

Q. Find determinant of matrix

$$\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

$R_1 \longrightarrow R_1 - R_2$ and $R_2 \longrightarrow R_2 - R_3$

$$\begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)$$

A,

$$|A| = |A^T|$$

Q. Find determinant of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+b \end{vmatrix}$$

$R_2 \longrightarrow R_2 - R_1$ and $R_3 \longrightarrow R_3 - R_1$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & b \end{vmatrix} = ab$$

standard results :-

(A) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 6 \end{vmatrix} = 20$

(B) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1+c \end{vmatrix} = abc$

Q. find the determinant of

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

$R_1 \longrightarrow \frac{R_1}{a}$. $R_2 \longrightarrow \frac{R_2}{b}$; $R_3 \longrightarrow \frac{R_3}{b}$.

$$= abc \begin{vmatrix} 1+\frac{1}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$R_1 \longrightarrow R_1 + (R_2 + R_3)$

$$=$$

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & 1+\frac{1}{b} & 1/b \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$C_2 \longrightarrow C_2 - C_1 \quad C_3 \longrightarrow C_3 - C_1$

$$= (abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \begin{vmatrix} 1 & 0 & 0 \\ 1/b & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$

$$= (abc)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

standard result:

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = 1\times1\times1\times1\left(1+\frac{1}{1}+\frac{1}{1}+\frac{1}{1}+\frac{1}{1}\right) = 5$$

Q. Find determinant:

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

$$\Delta = 0(2-1) - 1(1-9) + 2(1-6)$$
$$= -2$$

or

$$R_3 \longrightarrow R_3 - 3R_2$$

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & -5 & -8 \end{vmatrix}$$

$$\Delta = 0 - 1(-8+10) -$$
$$= -2$$

or

$$\begin{array}{ccccc} 0 & 1 & 2 & 0 & 1 \\ 1 & 2 & 3 & 1 & 2 \\ 3 & 1 & 1 & 3 & 1 \end{array}$$

$\Delta = \Sigma$ product of 1st diagonal elements $-$ Σ product of 2nd diagonal elements.

$$\Delta = 0 + 9 + 2 - 12 - 0 - 1$$

(1) $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = \begin{matrix} 1 & 2 & 2 & 1 & 2 \\ 2 & 1 & 2 & 2 & 1 \\ 2 & 2 & 1 & 2 & 2 \end{matrix} = 1 + 8 + 8 - 4 - 4 - 4 = 4$

(2) $\begin{vmatrix} 1 & 2 & 5 \\ 3 & 1 & 4 \\ 1 & 1 & 2 \end{vmatrix} = \begin{matrix} 1 & 2 & 5 & 1 & 2 \\ 3 & 1 & 4 & 3 & 1 \\ 1 & 1 & 2 & 1 & 1 \end{matrix} = 2 + 8 + 15 - 5 - 4 - 12 = 4$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{adj.\ A}{\Delta}$$

$$= \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \qquad = 1\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Inverse matrix exists only for non-singular matrix.

Adjoint of Higher order matrix is the transpose of the co-factor matrix.

Minor of element:

The minor of an element in square matrix is the determinant of square sub matrix in which the row and the column of particular element lies to be deleted.

$$A = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

$$\text{Minor of } a_{12} = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = (1-9) = -8$$

$$\text{Co-factor of an element} = (-1)^{i+j}\ \text{Minor of element}$$

$$= (-1)^{1+2} \times (-8)$$

$$= 8$$

i - no. of row & j - no. of column.

10/6

Chemical reaction Engineering

→ to design a reaction vessel (reactor)

→ Type of reactor (mode of operation)

Volume / size of reactor

Chemical reaction

formation & / breaking of new & old bonds resp.

Homogeneous
Single phase reaction
all Gas phase
or all liquid phase

Heterogeneous
more than one phase
S-G rxn
L-G rxn
Catalytic reaction

* fast - explosion
* slow - radioactive decay

• $1N_2 + 3H_2 \rightleftharpoons 2NH_3$ (Reversible & exothermic)
(Haber's process)

Contact reaction exothermic

1, 3, 2 → stoichiometric co-efficients : co-efficients of reactants or products

* stoichiometric cofficients of a chemical rxn represents moles, molecules, or volume (for gases rxn)
* the stoichiometric cofficient tells us about how the chemical reaction will proceed. (puts no restrictions on how much it should be taken)

$$1N_2 + 3H_2 \rightleftharpoons 2NH_3$$

Conservation of mass is valid in chemical rk^n

$t=0$	10 moles	20 moles	0
$t=t_1$ (1mole reacted)	9 moles	17 moles	2 moles
mass	28 gm	6 gm	34 gm
$t=t_2$	8 mols	14 mols	4 moles
$t=t_x$	4 mols	2 mols	12 moles
$t=t_y$	$4-2/3$	0 mols	$12+4/3$

rates is $\propto C_A$.

time increases as next rk^n happens

In reality no reaction goes to completion, it stops before which is decided by thermodynamics.

completion.

The reactant which get consumed first. is called the limiting reactant, while the other one is called excess reactant.

to find the limiting reactant, we have to assume that the reaction goes to completion.

to find the limiting reactant, we will divide the ~~initial~~ initial number of moles of reactants by their respective stoichiometric co-efficients

The reactant which gives lesser value is limiting ~~rea~~ reactant (2)

All this stoichiometric calculation in a reaction is done on the basis of the limiting reactant.

Stoichiometric proportion - Reactants are said to be in " if the ratio of the initial moles of the reactants is same as the ratio of the corresponding stoichiometric cofficients.

$$N_2 + 3H_2 \longrightarrow 2NH_3$$

	N_2	$3H_2$	
t=0	10 mole	10 mole	✗
t=0	10 mole	30 mole	✓
t=0	10 Kg	30 Kg	✗ (no because always it is done on basis of ~~mass~~ mole)

we can convert it into mol.

$$\frac{10}{28\ Kg/Kmol} \qquad \frac{30}{2\ Kg/Kmol} \quad ✓$$

If these are in stoichiometric proportion than both we get over at same time & either both can be limiting or none.

reversible $aA + bB \rightleftharpoons cC + dD$

$$K_C = \frac{[C]_e^c\,[D]_e^d}{[A]_e^a\,[B]_e^b} \quad \text{(are taken at equilibrium)}$$

Gate 2017

Q) The reversible reaction of tertiary butyl alchohol & ethanol to give ethyl teriatu butyl ether is given by.

$$TBA + EtOH \longrightarrow ETBE + H_2O$$

the equilibrium constant for this reaction is equal to 1

Initially 74 gm of TBA is mixed with 100 gm of aq sol^n containing 46% ethanol by wt

Given → $MW_{TBA} = 74$

$M.W_{EtOH} = 46$

$MW_{ETBE} = 102$

the mass of ETBE at eq^n

	TBA	+ EtOH	→ ETBE	+ H_2O
t=0	74 gm	46% of 100 g ~~100 gm~~	0	
t=0	1 mole	1 mol	0	$\frac{54}{18} = 3$ mol
t=teq.	$1-x$	$1-x$	x	$3+x$

$$1 = \frac{x\,(3+x)}{(1-x)(1-x)}$$

$$i = \frac{3x + x^2}{1 - 2x + x^2}$$

$1 - 2x + x^2 = 3x + x^2$

$x = 0.2$ mol

← In this question volume is constant so in place of concⁿ we can take moles.

<u>at equilibrium</u>

0.2 mols of E.TBE

∴ Mass of ETBE $= 0.2 \times 102$
$= 20.4$ gm

mass of $H_2O = 3.2 \times 18$
$= 57.6$ gm

$aA + bB \rightarrow cC + dD$

* <u>conversion</u> - it is only defined ~~only~~ for reactants and never for products. Conversion of a reactant A is denoted by X_A

$$X_A = \frac{\text{moles of A reacted}}{\text{moles of A fed}}$$

$$= \frac{\overset{\text{Initial}}{N_{A_0}} - \overset{\text{final (left)}}{N_A}}{N_{A_0}}$$

$$X_A = 1 - \frac{N_A/t}{N_{A_0}/t} \leftarrow \boxed{X_A = 1 - \frac{N_A}{N_{A_0}}} \rightarrow \text{for batch}$$

for continuous reactor ← $\boxed{X_A = 1 - \frac{F_A}{F_{A_0}}}$ → molar flow rate

It can also be expressed as a %. for solving numerical problems, we should always use the fractional value of conversion

For reporting the final answer, we should read the question and according to that conversion should be reported.

$$aA + bB \rightarrow cC + dD$$

$t=0$ $\quad N_{A0} \quad N_{B0} \quad N_{C0} \quad N_{D0}$

let us suppose conversion of A is known & it is X_A

(A here is limiting agent)

$$N_A = N_{A0}(1 - X_A)$$

moles of A reacted $= N_{A0} X_A$

B reacted $= \frac{b}{a}$ (A reacted)

$$= \frac{b}{a}(N_{A0} X_A)$$

$$N_B = N_{B0} - \frac{b}{a}(N_{A0} X_A)$$

$$N_C = N_{C0} + \frac{c}{a}(N_{A0} X_A)$$

$$N_D = N_{D0} + \frac{d}{a}(N_{A0} X_A)$$

relationship b/w X_A & X_B

$$N_B = N_{B0}(1 - X_B)$$

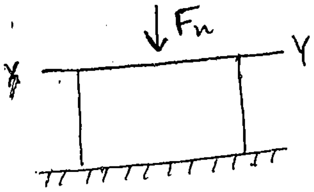
$$\cancel{N_{B0} - \frac{b}{a} = N_{B0}(1 - X_B)}$$

$$N_{B0} - \frac{b}{a}(N_{A0} X_A) = N_{B0}(1 - X_B)$$

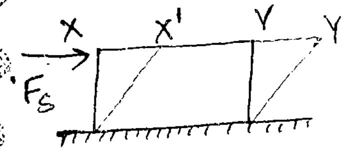
$$\boxed{X_B = \frac{b}{a} \frac{N_{A0}}{N_{B0}} X_A}$$

17/9 # Fluid Mechanics (1)

Normal forces → ||el to the cross-sectional area of the object.
It will change only the dimensions not the shapes



Shear forces → ||el to the cross section area of the object.
It will change only the shape not the dimension of the object.



fluid - It's a substance which deforms continuously under the action of shear forces i.e. fluid can't resists shear. The continuous rate of deformation is termed as flow.

Water, air - elastic behaviour is present in normal force sense and not in shear force sense.
It has different elasticity & diff deformation rate.

Motion - The velocity of all the particles is same & its just change in displacement

flow — Velocity of all particles are different & its layer by layer displacement.

* Absolute Vacuum cannot be ~~as~~ a system, but vacuum can be considered as a system.

System - Its an amount of matter whose physical characteristics are under observation / investigation

Property - Its the physical characteristics of a system which defines the state of the system
eg - P, T, etc

Intensive property - They are those which are independent of mass eg - every specific property, P, T

If a property is defined at a point then its intensive property & it doesn't need bulk to define it

Extensive property - The property will be extensive property if it depend upon mass (or) if it requires a bulk to define it. eg - weight, volume, entropy, enthalpy.

Physical properties of fluid

(1) Mass density (ρ) : density of water is 1000 Kg/m^3 only @ 4.4°C & 1 atm P

density of Hg is 13600 kg/m^3 only @ 25°C & 1 atm P.

Its the amount of mass of fluid occupied in unit volume at the given state of the fluid

$\rho = \frac{m}{V}$, kg/m^3

Note
density of water will always decrease no matter if you increase or decrease the temp (i.e) why the ice floats on water.

② Specific volume (v):
Specific volume is defined for gaseous & not liquid.
Specific volume is defined as volume occupied by unit mass of fluid.
Its defined as volume occupied by unit mass of fluid.

$$v = \frac{V}{m} = \frac{m^3}{kg}$$

Note
The concept of specific volume is useful for compressible fluids (i.e) gaseous because, gases can occupy different volume for the same amount of mass at a different given state.

③ Specific weight / weight density (w):
Its an amount of weight of the fluid occupied in unit volume at the given state & in the given field (gravitational field)

$$\boxed{w = \frac{W}{V} = N/m^3} = \frac{mg}{V} = \rho g.$$

- Specific weight of water = $9.81 \times 1000 = 9810\ N/m^3$
@ 4.4°C & 1 atm P.

④ **Specific gravity**

Its the ratio of mass density (or) weight density of unknown fluid to mass ρ/ weight ρ of some standard fluid.

$$s.g = \underset{\substack{\downarrow \\ \text{denser} \\ \text{than}}}{\frac{\rho}{\rho_{st}}} \quad \text{or} \quad \underset{\substack{\downarrow \\ \text{heavier} \\ \text{than}}}{\frac{w}{w_{st}}}$$

For liquids, standard fluid is water @ 4.4°C & 1 atm P.

For gases, std fluids is air & $\rho_{air} = 1.23 \frac{kg}{m^3}$ @ 25°C, 1 atm.

NOTE:

- for std fluids, the specific gravity is 1
- Now, if for some fluid, the specific gravity is less than 1 than that fluid will be lighter & less dense compared to std fluid & it will float over std fluid.
  - eg - water & ice
- If the specific gravity of unknown fluid is greater than 1 then that fluid is denser & heavier than std fluid & will sink down in std fluid. eg - Mercury

Q) 2 L of petrol of weight 14 N. Cal. sp. weight, mass ρ, sp. volume & sp. gravity. (3)

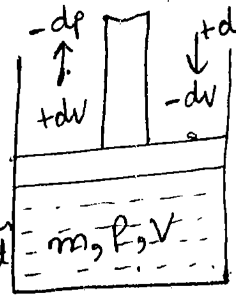
$\rightarrow S.g = \frac{\rho_L}{\rho_w}$ $\rho_l = \frac{14/9.8}{0.002} = 714.2857$ kg/m^3

$S.g = \frac{714.2857}{1000} = 0.7142$ The petrol will float over water

$\therefore$ Sp. vol. $= \frac{1}{\rho} = \frac{1}{714.2857} = 0.0014$

Sp. weight $= \frac{W}{V} = \frac{14}{0.002}$ $= 7000$ N/m^3

(5) <u>Bulk modulus of elasticity (K)</u> & <u>coefficient of compressibility (β_c)</u>



Mass remains constant here.

Hooke's law $\rightarrow$ stress $\propto$ strain

Stress $\rightarrow$ 2nd order tensor quantity (diff value of diff directions)

Pressure $\rightarrow$ Scalar quantity

Stress $\rightarrow$ Induced quantity

Pressure $\rightarrow$ Applied quantity

Zero order tensor is scalar (same value in all directions)

1st order tensor $\rightarrow$ Vector

U can induce stress by applying pressure but not vice-versa.

Moment of Inertia $\rightarrow$ 2nd Order Tensor (when values are diff at diff mutually in directions

$\sigma_N \propto \varepsilon_V$

$dp \propto \varepsilon_V$

$dp \propto -\frac{dv}{V}$

$dp = -k\frac{dv}{V}$ $\quad\therefore \boxed{k = dp \Big/ \left(\frac{-dv}{V}\right)}$

Under normal forces water acts as an elastic material.

$m = C$

$m = \rho \cdot V$

$dm = \rho dv + V d\rho = 0$

$\rho dv = -V d\rho$ $\quad\therefore \boxed{k = dp \Big/ \left(\frac{d\rho}{\rho}\right)}$

$-\frac{dv}{V} = \frac{d\rho}{\rho}$ $\quad$ 'k' unit $\rightarrow$ Pa (or) N/m^2

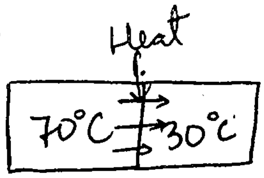
- 'k' value is greater than the pressure to be applied for a specific change in volume should be greater.
- 'k' value is greater than the fluid behaves more elastic.
- 'k' value is less than it for a greater amt of pressure also it will cause only less volume change.

→ From the experiment & by using Hooke's law. we can say that the pressure change will be prop. to change in volume but opposite in nature (i.e) if pressure ↑ es volume ↓

20/6.

Heat transfer

① Prof. CP Arora ⑪
② " RC Sachdeva
③ " DS Kumar
④ " Domkundwar.



When two bodies which are in contact, than the transfer which occurs is called
* **heat** (the energy which flows by virtue of temperature difference)

Water flows from higher potential to lower potential till they are at equal potential.

* Temperature - thermal potential of system responsible for heat transfer

Unit of heat -

1 gm of water × 1°C = 1 Calorie — CGS

1 kg. of water × 1°C = 1 Kilo Calorie
1 Kcal

1 lb of water × 1°C = 1 CHU
Centigrade Heat Unit

1 lb of water × 1°F = 1 B.Th.U (British Thermal Unit)
B.T.U or Btu

Work & Heat (similar type of energy in transient state
↓
KNm & Kcal

4.1868 KJ → 1 Kcal

/ * KJ & Kcal

Kcal is a fundamental unit of Heat

KJ is a derived unit of Heat

* Modes of Heat transfer

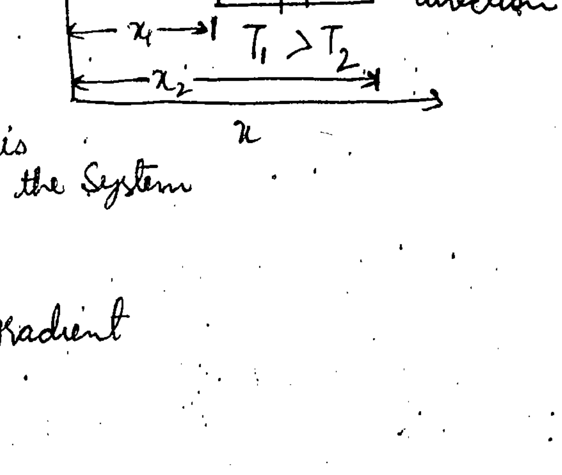
1) Conduction — 10 hrs
2) Convection — 20 hrs
3) Radiation — 10 hrs.

6 hrs exchangers

4 hrs change in phase

Conduction

Plane wall



* assumptions

1. One-dimensional heat transfer
2. Steady state heat transfer (temperature does not change with time)
3. k remains same (it does ~~not~~ vary with temperature)
4. No heat generation
5. Isothermal surfaces
6. No heat is retained in the System

Fourier's law of heat Conduction

Heat flux $\propto$ Temperature gradient

$$\frac{Q}{A} \propto \frac{dT}{dx}$$

normal area to the direction of heat transfer

*

$$\frac{Q}{A} = -k\frac{dT}{dx}$$

k → Constant of proportionality

k → Thermal conductivity of material

$$\boxed{Q = -kA\frac{dT}{dx}}$$

$$dx = -\frac{kA}{Q}\,dT$$

Integrating, $\displaystyle\int_{x_1}^{x_2} dx = -\frac{kA}{Q}\int_{T_1}^{T_2} dT$

$$x_2 - x_1 = -\frac{kA}{Q}(T_2 - T_1)$$

$$= \frac{kA}{Q}(T_1 - T_2)$$

$$\therefore\; Q = \frac{kA(T_1 - T_2)}{(x_2 - x_1)} = \frac{kA(T_1 - T_2)}{\Delta x}$$

$$Q = \frac{T_1 - T_2}{\Delta x / kA}$$

$I = V/R$

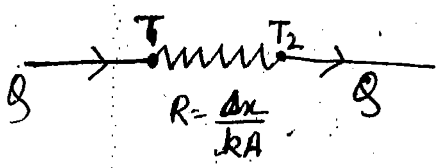
Current = $\dfrac{\text{electrical potential diff.}}{\text{electrical resistance}}$

$$k = \frac{Q\,\Delta x}{(T_1 - T_2)\,A} = \frac{J/s \times m}{°C \times m^2}$$

$\boxed{J/s \rightarrow \text{Watt}}$

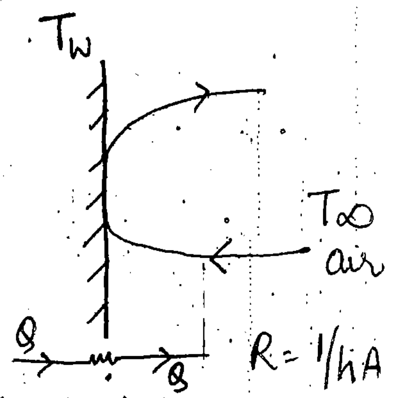
$$= \boxed{W/°Cm}$$

$$= \boxed{W/°Fm}$$



$$\boxed{R = L/kA}$$

Convection



let $T_W > T_\infty$

at the surface the velocity of air is zero and as we move slowly forward velocity increases.

Newton - Rickman's law

Heat flux $\propto$ Temp. difference

$$\frac{Q}{A} \propto (T_W - T_\infty).$$

$$\frac{Q}{A} = h_c (T_W - T_\infty)$$

h_c → constant of proportionality
→ Convective heat transfer co-efficient
→ film co-efficient

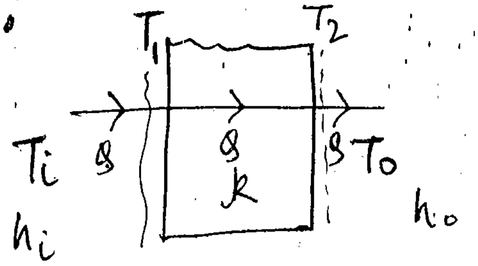
$$\boxed{Q = h_c A (T_W - T_\infty)}$$

(*) Units $= \frac{J/s}{m^2 \cdot {}^\circ C} = W/m^2 {}^\circ C$
$= W/m^2 {}^\circ F$
$= W/m^2 K$

$$Q = \frac{T_W - T_\infty}{1/hA}$$

$1/hA$ → thermal resistance

(3)



$q = \cancel{h_i (T_i - T_1)}$

$$q_1 = h_i A (T_i - T_1) \Rightarrow q_1 = \frac{T_i - T_1}{1/h_i A}$$

$$\frac{q_1}{h_i A} = T_i - T_1$$

$$q_2 = \frac{(T_1 - T_2)}{l/kA}$$

$$T_1 = T_i - q_1/h_i A$$

$$q_3 = h_o A (T_2 - T_o)$$

$$q = q_1 = q_2 = q_3$$

$$\frac{q_3}{h_o A} = T_2 - T_o$$

$$T_2 = T_o + \frac{q_3}{h_o A}$$

$$\therefore q = q_1 = q_2 = q_3$$

$$= h_i A \left(T_i - \left(T_i - \frac{q_1}{h_i A}\right)\right) = \frac{T_i - \frac{q_1}{h_i A} - \left(T_o + \frac{q_3}{h_o A}\right)}{L/kA}$$

$$= h_o A \left(T_o + \frac{q_3}{h_o A} - T_o\right)$$

all q are same so solve & get / answer.

$$q = h_i A \left(\frac{q_1}{h_i A}\right) = T_i - \frac{q_1}{h_i A} - \left(T_o + \frac{q_3}{h_o A}\right)$$

$$= h_o A \left(\frac{q_3}{h_o A}\right)$$

$$Q = \frac{T_i - T_1}{\frac{1}{h_i A}} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_0}{1/h_0 A}$$

$$T_i - \cancel{T_1} = \frac{Q}{h_i A}$$

$$\cancel{T_1} - \cancel{T_2} = \frac{Q L}{kA}$$

$$\cancel{T_2} - T_0 = \frac{Q}{h_0 A}$$

$$T_i - T_0 = Q\left[\frac{1}{h_i A} + \frac{L}{kA} + \frac{1}{h_0 A}\right]$$

$$Q = \frac{T_i - T_0}{\frac{1}{h_i A} + \frac{L}{kA} + \frac{1}{h_0 A}} = \frac{V}{\Sigma R_t} \longrightarrow (1)$$

Overall heat transfer co-efficient $(U) \rightarrow$ combining overall co-efficient of conduction + convection

$$Q = UA(T_i - T_0)$$

$$Q = \frac{T_i - T_0}{1/UA} \quad -(2)$$

$$\boxed{\text{Unit} = W/m^2\,{}^\circ C}$$

compare (1) + (2)

$$\frac{1}{U\cancel{A}} = \frac{1}{h_i \cancel{A}} + \frac{L}{k\cancel{A}} + \frac{1}{h_0 \cancel{A}}$$

$$U = \frac{1}{\frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_0}}$$

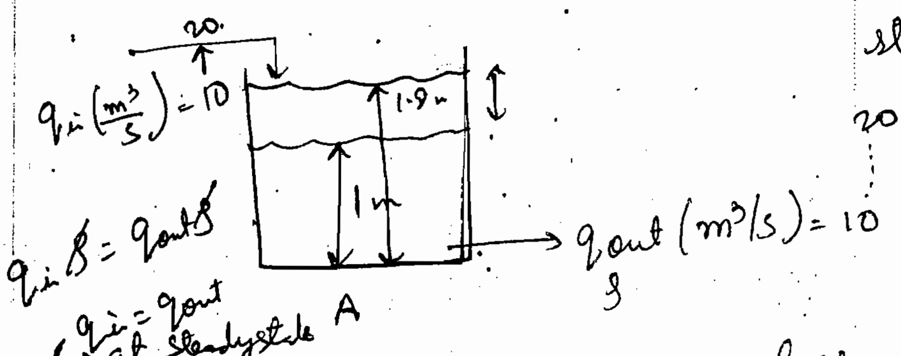
Process Dynamics & Control

- Process Dynamis
- Control Strategies
- ** *Stability Analysis
- Stability Analysis
- Advanced process control
- Instrumentation

Process dynamics

- It is the study of behaviour of a process/system when it is going from one steady state to another steady state
- A process goes from one steady state to another steady state when it is disturbed by some disturbance in a specific manner.

It changed from one steady state to another in some time after q_{in} is disturbed.



How to Understand the behaviour of process

We Will write the model of the process

Model - It means a set of mathematical equations which govern the process.

This mathematical equations are written by writing the conservation laws for the process.

Apart from the conservation equations the mathematical model also consists of constitutive relationships

Constitutive relationships

eg = $PV = nRT$, $Q = UA\Delta T_{LMTD}$, $(-r_A) = kC_A^n$

- Mathematical model of a process is of two types :-

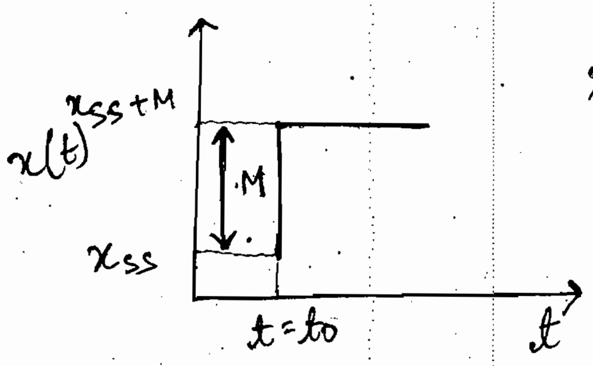
  i) Steady state model

  ii) Unsteady state model

- If we study the steady state behaviour than we will write steady state model and if we are interested in unsteady state behaviour than we will write unsteady state model.

Input / Disturbance / forcing functions

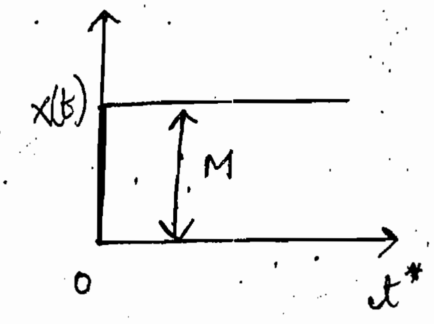
① Step input: when the change is sudden and it continues for infinite time.



$$x(t) \begin{cases} x_{ss} & t < t_0 \\ x_{ss}+M & t \geq t_0 \end{cases}$$

In Process dynamics, we are interested in the value of change and not the absolute values

②

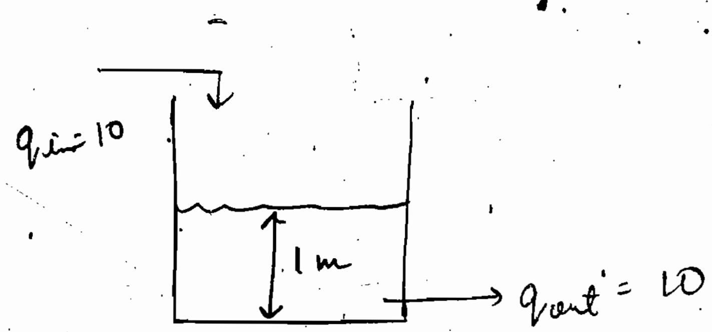


$t - t_0 = t^*$

$x(t) - x_{ss} = X(t)$

$$X(t) \begin{cases} 0 & t^* < 0 \\ M & t^* \geq 0 \end{cases}$$

In process dynamics we talk in terms of deviation variable $X(t)$ and not in terms of actual variables $x(t)$. In process dynamics $t=0$ is that time at which the process is disturbed.



t	$y(t)$	y_{ss}	$Y(t)$
0	1 m	1 m	0
t_1	1.2 m	1 m	0.2 m
t_2	1.3 m	1 m	0.3 m
⋮	⋮	⋮	⋮

$x_{ss} = 10$

$x(t) = 20$

$X(t) = 20 - 10 = 10$

$y_{ss} = 1$ m

$y(t) = 1$

$y(t) \rightarrow$ actual output variable

$y_{ss} \rightarrow$ steady state output value

$Y(t) \rightarrow$ deviation output variable

Note:

** $\boxed{Y(t)_{t=0} = 0}$

Deviation variable = actual variable − Initial steady state

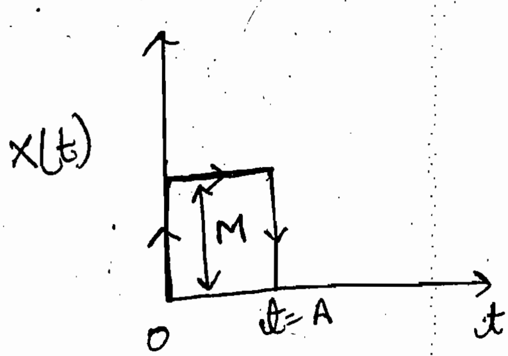
- Step input can be positive or negative in nature.
- Step input of magnitude M is written as $X(t) = Mu(t)$ → If +ve step input

for negative step input of magnitude M

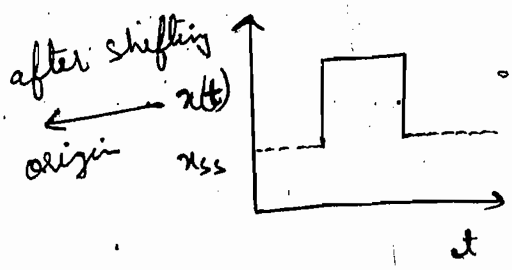
Δ $X(t) = -Mu(t)$

- Step Input of unit magnitude is called Unit Step function (or) unit step input or its denoted by $u(t)$.

② Pulse/ rectangular pulse.



<u>actual variable</u>



$$X(t) \begin{cases} 0 & t < 0 \\ M & 0 \le t \le A \\ 0 & t > A \end{cases}$$

- The above function can be treated as a combination of two step functions.

A pulse function is a combination of one positive step & negative step (one +ve step @ $t = 0$ & one -ve step at $t = A$)

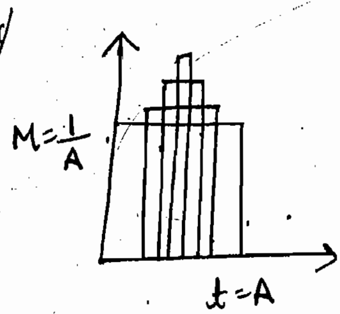
$$X(t) = \underbrace{M u(t)}_{t=0} \underbrace{-}_{\text{-ve sign due to negative step}} \underbrace{M u(t-A)}_{t=A}$$

- If the area of the is equal to 1 than it is called unit pulse input.

(B)

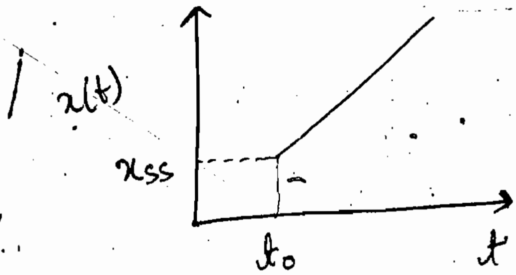
③ Unit Impulse

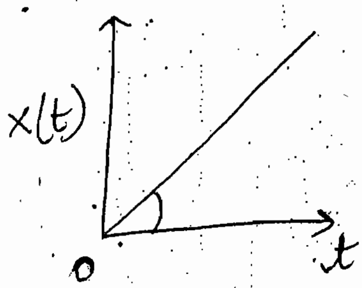
In a unit pulse function if $A \to 0$ such that the area remains 1, the function becomes unit Impulse function



A unit Impulse function is denoted by $X(t) = \delta(t)$

④ Ramp Input / Ramp function

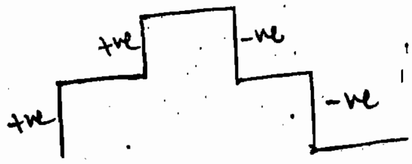




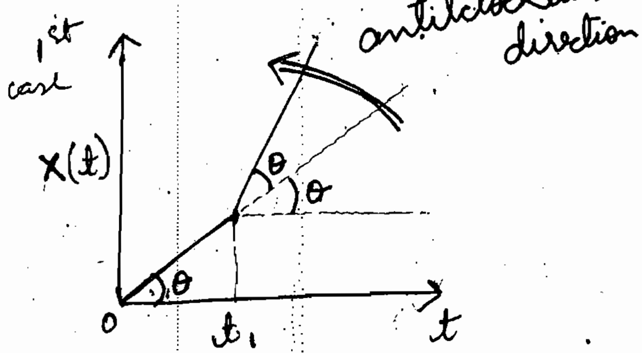
Slope = M

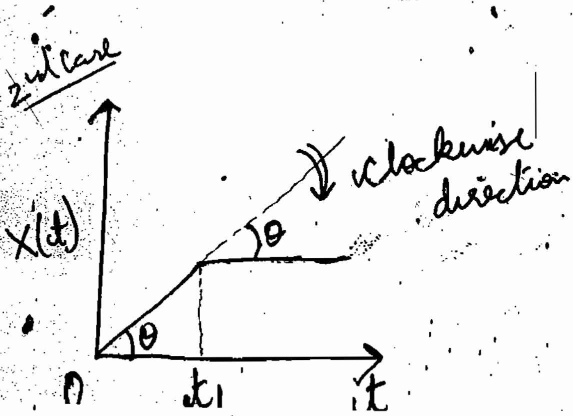
$X(t) = M\,t\,u(t)$

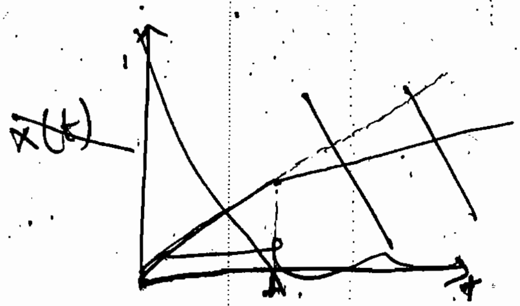
Step + Step



ramp + ramp







When a ramp input is given to another ramp input, than it leads to rotation. The direction of rotation depends upon the sign of the Input.

If +ve ⇒ anticlockwise

−ve ⇒ clockwise

for 2nd case

$$X(t) = Mt\,u(t) - M(t - t_1)\,u(t - t_1)$$

$u(t-t_1)$ & $u(t)$ ⇒ Its just a representation

⑤ Sinusoidal Input / function

$$X(t) = A \sin \omega t$$

ω → frequency {rad/s}.

A ↳ amplitude

Laplace Transform

Laplace transform of a function $f(t)$ is defined as

$$\boxed{L\{f(t)\} = F(s) = \int_0^{\infty} e^{-st} f(t)\,dt}$$

① $L\{1\} = \frac{1}{s}$ & $L\{k\} = k/s$

② $L\{t^n\} = \frac{n!}{s^{n+1}}$; n should be +ve integer

Methods to Calculate depreciation

Book Value of the Equipment - defined as the value of equipment after the end of an year (financial year end or start of finacial year)
It is denoted by V_a

Service life - The time period upto which the use of the property is economically feasible. eg - If the heat exchanger (equipment) is purchase for getting a fixed amount of heat exchange, we will not replace it with new one till the time it is saving the money, but when the use is becoming costly than it gets replaced with new one. That time of use is known as service life.
It is denoted by n

Salvage value or Scrap Value -
The money obtainable by the selling of the equipment over and above any charges (uninstalling & transportation cost) is known as salvage value or scrap value.
It is denoted by V_s
If V_s is non-zero than this is known as Salvage Value & if V_s is zero than it is known as Scrap Value.

**

Depriciation accounting has no relations with the real sale or purchase. This whole concept is made just for the sake of balance sheet.

Straight line method

assumptions.

- The depreciation amounts for the subsequent years are same.

Time	Book Values
0	5L → V_0
1	4L → V_1
2	3.5L → V_2
3	1.5L → V_3
4	1L → V_4
⋮	⋮
n	V_s

n → Service life

V_s → Salvage or Scrap

$$D_1 = V_0 - V_1$$
$$D_2 = V_1 - V_2$$
$$D_3 = V_2 - V_3$$
$$D_4 = V_3 - V_4$$
$$\vdots$$
$$D_n = V_{n-1} - V_s$$

$$\sum_{i=1}^{n} D_i = V_0 - V_s$$

In the case of straight line, $D_1 = D_2 = D_3 = \dots = D_n$

and that's why

$$nD = V_0 - V_s$$

** $$\boxed{D = \frac{V_0 - V_s}{n}}$$

depriciation

** $$\boxed{V_a = V_0 - aD}$$

Book Value

* Straight line method is valid for both Salvage + Scrap. ③

8) Heat Ex is to be installed in a industry at an initial investment of rs 10 lakh. The service life of equipment was assumed to be 10 years + Salvage value approximate to be 2 lakh. If straight line depreciation method is used for depreciation account then Cal ~~the~~ depreciation ~~to be~~ during the second year of plant operation + book value after the end of third year operation

$$D = \frac{10-2}{10} = 80 \text{ thousand} = .8 \text{ lakh}$$

~~D = (10-2)/3 = 8/3 = 2.66~~

$V_a = 10 - 3 \times .8$
$= 7.6$ lakh

depreciation is always same for every year but book value is not same for every year.
(in straight line method)

Declining balance method

assumptions

depreciation amounts for the subsequent years are not same but fixed % factors are same.

Time	Book Values	fixed % factor	depreciation amount
0	5 L	10% ↓	$D_1 = 50K$
1	4.5 L	10% ↓	$D_2 = 45K$
2	4.05 L		

$$4.5L = 5L - 10\% \text{ of } 5L.$$

$$V_1 = V_0 - fV_0$$

$$V_1 = V_0(1-f)$$

$$4.05L = 4.5L - 10\% \text{ of } 4.5L$$

$$V_2 = V_1 - fV_1$$

$$= V_1(1-f)$$

$$V_2 = V_0(1-f)^2.$$

★★ $$\boxed{V_a = V_0(1-f)^a.}$$

★★ $$\boxed{D_a = V_{a-1} \times f}$$

V_0, V_s + n these three data points is always avialable to us and it is supplied by the manufacturer. These data helps us in comparing different alternatives and also used in the depriciation accounting purposes.

$$V_s = V_0(1-f)^n$$

$$\frac{V_s}{V_0} = (1-f)^n$$

$$\left(\frac{V_s}{V_0}\right)^{\frac{1}{n}} = (1-f)$$

★★ $$\boxed{f = 1 - \left(\frac{V_s}{V_0}\right)^{1/n}}$$

* declining-balance method is valid only for salvage and in the case of scrap the depriciation is 100 % in the first year which is not possible. (4)

$V_0 = 10$ lakh.

$V_S = 2$ lakh.

$$f = 1 - \left(\frac{2}{10}\right)^{1/10}$$

$\therefore f = 0.148$

$V_3 = 10(1 - 0.148)^3$
$= 6.18$ lakh.

$D_3 = 7.259 \times 0.148$
$= 1.074$ lakh.

$V_{a-1} = 10(1 - 0.148)^a$
$V_2 = 7.259$ lakh.

$V_1 = 10(1 - 0.148)^1$
$= 8.52$

$D_2 = 1.26$ lakh

Double declining method

assumption

depriciation amounts for the subsequent years are not same but fixed % years are same.

** $$\boxed{V_a = V_0 . (1-f)^a}$$

** $$\boxed{D_a = V_{a-1} \times f}$$

** $$\boxed{f = 2/n}$$

9) $f = 2/10 = 0.2$

~~V_a~~ $V_3 = V_0(1-f)^3$

$V_3 = 10(1-0.2)^3$

$= 5.12$ lakh.

$V_2 = 10(1-0.2)^2$

$= 6.4$ lakh.

$V_1 = 10(1-0.2)^1$

$= 8$ lakh

$D_3 = 6.4 \times 0.2$

$= 1.28$ lakh.

$D_2 = 8 \times 0.2$

$= 1.6$ lakh.

Sum ~~Some~~ of the years digits method

depreciation amounts for the subsequent years are not same.
Also, fixed % ptage factor is not same.

$$\boxed{D_a = \frac{n - a + 1}{\Sigma n}\{V_0 - V_s\}} \qquad \boxed{\Sigma n = \frac{n(n+1)}{2}}$$

Valid for salvage as well as scrap.

$$\cancel{\frac{10 \times 8}{2}} \quad \frac{\overset{5}{\cancel{10}} \times 11}{\cancel{2}}$$

$$D_2 = \frac{10 - 2 + 1}{55 \cancel{\ldots}} \times (10 - 2)$$

$= \cancel{10.6\ \text{lakh}}\ 1.3$ lakh.